

# Pseudo-Goldstone Modes in Isospin-Asymmetric Nuclear Matter

Thomas D. Cohen\*

*Department of Physics and Institute for Nuclear Theory, University of Washington,  
Seattle, WA 98195, USA*

Wojciech Broniowski

*H. Niewodniczański Institute of Nuclear Physics, PL-31342 Cracow, POLAND*

## Abstract

We analyze the chiral limit in dense isospin-asymmetric nuclear matter. It is shown that the pseudo-Goldstone modes in this system are qualitatively different from the case of isospin-symmetric matter.

---

\*On leave from Department of Physics and Astronomy, University of Maryland, College Park, MD 20742

The concept of mesons modified by the nuclear medium has been used for many years [1]. It has been generally assumed that although the properties of pions and kaons (mass, decay constant, coupling constants, etc.) are changed by the presence of the medium, their *pseudo-Goldstone nature* is preserved.<sup>1</sup> In the vacuum, the masses of pions or kaons are known to be proportional to the square root of the current quark masses, according to the Gell-Mann–Oakes–Renner (GMOR) relation [2]. However, in nuclear matter this need not be the case.

In this paper we show, that the nature of *charged* pionic excitations in a dense *isospin-asymmetric* medium, or kaonic excitations in a non-strange medium, is radically different than what is seen in the vacuum results. With no dynamical assumptions other than the fact the current quark masses are sufficiently light, and that expectation values of physical operators do not diverge, one can show that in such a medium there must exist at least one pseudo-Goldstone mode with an energy that scales with the average current quark mass as  $\overline{m}^d$  where  $d \geq \frac{2}{3}$ . This is in sharp contrast with the case of the vacuum or isoscalar matter, where the energy of the Goldstone modes goes as  $\overline{m}^{1/2}$ .

Moreover, if one makes highly plausible assumptions about the behavior of the chemical potential for the system as one varies the quark masses, then one can show even more peculiar behavior. In the vacuum one has pairs of charged pseudo-Goldstone excitations ( $\pi^+\pi^-$  or  $K^+K^-$ ). As one goes to a dense isospin-asymmetric medium (such as dense neutron matter), however, given these assumptions one can show that only one member of this pair survives as a pseudo-Goldstone mode. In addition, the excitation of the surviving pseudo-Goldstone boson is proportional to the current quark mass  $m_q$  itself, rather than to  $\sqrt{m_q}$ , as in the vacuum case. Specifically, we find that in the chiral limit in neutron matter there exist 1) a pseudo-Goldstone mode with quantum numbers  $\pi^+$  with excitation energy  $\sim m_q$ ,

---

<sup>1</sup>A mode can be identified as a pseudo-Goldstone mode, if its excitation energy vanishes in the chiral limit of zero quark mass.

- 2) pseudo-Goldstone mode with quantum numbers  $\pi^0$  with excitation energy  $\sim \sqrt{m_q}$ , and
- 3) there are no pseudo-Goldstone modes with quantum number of  $\pi^-$ .

Our analysis is similar in many respects to that of Ref. [3], however that work implicitly assumes that the medium has low density. Mathematically, this equivalent to expanding in density before expanding in the quark mass. This is clearly appropriate for mesonic atoms, where the meson feels only the tail of the nucleus at a fraction of the nuclear saturation density,  $\rho_0$ . Here we are interested in studying sufficiently dense systems so that it becomes appropriate to consider *chiral limit with the density kept constant*. We will present some model estimates indicating that this situation may be realized in nature if the density of matter is of the order of a few  $\rho_0$ . We will also demonstrate explicitly for a simple model that the chiral and low (isovector) density limits do not commute: an expansion in the small parameter associated with one limit is singular in the expansion parameter associated with the other limit. Thus, physics associated with the two limits will be quite different.

The paper is organized as follows: First, we derive our general result of existence of unusually soft pseudo-Goldstone modes in nonsymmetric nuclear medium. Next, we present a simple model illustrating that behavior and give some numerical estimates of where we might see these unusually soft modes. Finally, we discuss the case of kaons in non-strange matter.

We find it useful to introduce the function  $\dim_\chi(X) = \lim_{\bar{m} \rightarrow 0} (\bar{m} \frac{d \log X}{d \bar{m}})$ , which we call the *chiral dimension* of the quantity  $X$ . If near the chiral limit  $X$  scales with the quark mass as a power law in  $\bar{m}$ , *i.e.* if  $X \sim \bar{m}^\alpha$ , then function  $\dim_\chi(X)$  extracts the power  $\alpha$ . For instance, in the vacuum  $\dim_\chi(m_\pi) = 1/2$  and  $\dim_\chi(F_\pi) = 0$ . The result is not modified by the presence of logarithms *i.e.*  $\dim_\chi(\bar{m}^\alpha \log^n \bar{m}) = \alpha$ .

The starting point of our analysis is similar as in the derivation of the standard GMOR relation. We examine the expression for the double commutator of the QCD hamiltonian density at point  $x = 0$ ,  $\mathcal{H}(0)$ , with the axial vector charges  $Q_5^a = \int d^3x J_{5,0}^a(x)$ , where  $J_{5,\mu}^a$  is the axial vector current. This expression can be evaluated directly through the use of commutation rules for the quark fields:

$$[Q_5^a, [Q_5^a, \mathcal{H}(0)]] = \bar{\psi}(0) \{ \lambda^a/2, \{ \lambda^a/2, M \} \} \psi(0) , \quad \text{any } a = 1, \dots, 8 , \quad (1)$$

where  $\lambda^a$  are the Gell-Mann flavor matrices,  $M = \text{diag}(m_u, m_d, m_s)$  is the quark mass matrix, and curly brackets denote anti-commutators. Next, we take the matrix element of both sides of the above operator relation in some physical state  $|C\rangle$ . In all of our applications this state will be taken to be *spatially uniform*. The matrix element of the RHS of Eq. (1) yields a linear combination of quark condensates of various flavors. For simplicity of notation, let us consider the case of two flavors, with  $m_u = m_d = \bar{m}$ . The generalization to 3 flavors is straightforward. We then obtain

$$\langle C | [Q_5^a, [Q_5^a, \mathcal{H}(0)]] | C \rangle = \bar{m} \langle C | (\bar{u}u(0) + \bar{d}d(0)) | C \rangle \equiv \bar{m} \langle \bar{q}q \rangle_C , \quad a = 1, 2, 3. \quad (2)$$

On the other hand, the LHS of the preceding equation may be evaluated by inserting a complete set of intermediate states, yielding a sum rule.

Since the system  $|C\rangle$  is spatially uniform, these intermediate states can be labeled by their three-momentum, and some additional label  $j$ . We denote them as  $|j, \vec{p}\rangle$ . It is convenient to measure the three-momentum,  $\vec{p}$ , and the energy,  $E_j$ , of the intermediate states *relative* to the ground state  $|C\rangle$  at rest. The total energy and the total three-momentum of the state  $|j, \vec{p}\rangle$  and of the ground state  $|C\rangle$  both form Lorentz four-vectors. Their difference is therefore also a four-vector, and the unity can be decomposed in the following Lorentz-invariant way:

$$1 = \sum_j \int \frac{d^3p}{(2\pi)^3} \frac{1}{2|E_j|} |j, \vec{p}\rangle \langle j, \vec{p}|. \quad (3)$$

Inserting Eq. (3) inside the LHS of Eq. (1), and using the fact that the ground state and the intermediate states are eigenstates of the hamiltonian  $H = \int d^3x \mathcal{H}(x)$ , namely  $H|C\rangle = E_C|C\rangle$  and  $H|j, \vec{p}\rangle = (E_C + E_j)|j, \vec{p}\rangle$ , we obtain the following relation:

$$- \sum_j \frac{E_j}{|E_j|} \left| \langle C | J_{5,0}^a(0) | j, \vec{p} = 0 \rangle \right|^2 = \bar{m} \langle \bar{q}q \rangle_C , \quad (4)$$

We may use the following operator relation

$$\partial^\mu J_{5,\mu}^a(0) = [Q_5^a, \mathcal{H}(0)] \equiv \bar{m} D^a(0), \quad D^a(0) = \bar{\psi}(0) i \gamma_5 \tau^a \psi(0) , \quad (5)$$

to relate the matrix elements of  $J_{5,0}^a$  to the matrix elements of  $D^a$ . We find

$$\langle C | J_{5,0}^a(0) | j, \vec{p} = 0 \rangle = -i \frac{\overline{m}}{E_j} \langle C | D^a(0) | j, \vec{p} = 0 \rangle. \quad (6)$$

Using Eq. (6) we may rewrite Eq. (4) in the equivalent form which is familiar from the derivation of the GMOR relation:

$$\sum_j \frac{\overline{m}}{|E_j|E_j} |\langle C | D^a(0) | j, \vec{p} = 0 \rangle|^2 = -\langle \overline{q}q \rangle_C. \quad (7)$$

Let us now recall the usual arguments leading from the preceding general result to the GMOR relation. In the GMOR case, the state  $|C\rangle$  is simply the vacuum. First, we note that all excitation energies  $E_j$  are positive, since the vacuum is, by definition, the lowest-energy state of the system. Therefore all components of the LHS are positive. Next, consider taking the limit  $\overline{m} \rightarrow 0$  on both sides of Eq. (7). The RHS is assumed to go to a nonzero constant as a result of the spontaneous chiral symmetry breaking, *i.e.*  $\dim_\chi(\langle \overline{q}q \rangle) = 0$ . On the LHS only the terms with pseudo-Goldstones can contribute. This is because  $\dim_\chi(\langle 0 | D^a(0) | j, \vec{p} = 0 \rangle) = 0$ , and thus only the states for which  $\dim_\chi(E_j) = 1/2$  can contribute to the sum rule in the chiral limit. In the vacuum,  $E_j$  is just the mass of the excited particle (pion). Furthermore, from isospin symmetry we have  $m_{\pi^0} = m_{\pi^+} = m_{\pi^-}$ .

In the present case our state  $|C\rangle$  is not the vacuum. Rather, we take  $|C\rangle$  to be the ground state of the system subject to a *constraint* which requires that the expectation value of some local (space-independent) operator  $c(x)$  (which commutes with  $H$ ) has a fixed value, *i.e.*  $\langle C | c(x) | C \rangle = c = \text{const.}$  In our case we are interested in medium with a given flavor density, and  $c(x) = \mu_u u^+ u + \mu_d d^+ d$  or  $c(x) = 3\mu_B(u^+ u + d^+ d) + \mu_{I=1}(u^+ u - d^+ d)$ , where  $\mu$ 's are the corresponding chemical potentials for up and down flavors, or alternatively for the baryon number density and the isovector density operator  $u^+ u - d^+ d$ . If  $c \neq 0$ , then we have no guarantee that  $E_j$  is positive definite. The operator  $J_{5,0}^a$  may connect to states with a lower energy.

There are two distinct cases to consider: 1) the operator  $c(x)$  commutes with the axial vector charges or 2) the operator  $c(x)$  does not commute with the axial vector charges. As

already noted in Ref. [4], if  $c(x)$  commutes with  $Q_5^a$  (or equivalently with  $D^a$ ), then the states with negative  $E_j$  do not contribute in the sum rule. This is simply because in this case  $D^a$  connects  $|C\rangle$  only to intermediate states with the same value of  $c$ , and, by definition,  $|C\rangle$  is the *lowest* energy state of a given value of  $c$ .

One physical situation where the constraint commutes with the  $D^a$  operators is the case of finite baryon density with zero isovector density (isospin symmetric nuclear matter). In this case the conclusions from the sum rule (7) are obtained analogously to the vacuum case, and, so long as  $\langle \bar{q}q \rangle_C \neq 0$  in the chiral limit, we conclude there must exist pseudo-Goldstone excitations with  $E_j \sim \sqrt{\bar{m}}$  with quantum numbers of  $\pi^0$ ,  $\pi^+$  and  $\pi^-$ .<sup>2</sup> Isospin symmetry causes the neutral and charged excitations to be degenerate in energy. Thus, from the point of view of the GMOR relation, symmetric nuclear matter behaves similarly to the vacuum.

Now we come to the main topic of our considerations. Take nonsymmetric nuclear medium, *i.e.*  $\rho_{I=1} \equiv \langle C | (u^+u - d^+d) | C \rangle \neq 0$ . The neutral pseudo-Goldstone excitation still behaves in the usual way, since the neutral axial vector charge commutes with the third isospin component of the isospin charge,  $Q_3^3$ . Hence, as already remarked in [4], even in nonsymmetric matter we have a neutral pseudo-Goldstone excitation such that  $E_{\pi^0} \sim \sqrt{\bar{m}}$ . The case of charged pionic excitations, however, is radically different for two reasons. The first is that the isovector constraint does not commute with the charged axial vector charges  $Q_5^1$  and  $Q_5^2$ . The second difference is related to the existence of a second sum rule which is trivially zero for an isoscalar medium but not for a medium with nonzero isospin density. This sum rule is derived rather easily. Using the fact that  $[Q_5^a, J_{5,0}^b(0)] = i\epsilon^{abc} J_0^c(0)$ , where  $J_\mu^a$  is the vector current, and inserting Eq. (3) inside the commutator, we obtain

$$\rho_{I=1} = \sum_{j_-} \frac{\bar{m}^2}{|E_{j_-}|E_{j_-}^2} |\langle j_-, \vec{p} | D^-(0) | C \rangle|^2 - \sum_{j_+} \frac{\bar{m}^2}{|E_{j_+}|E_{j_+}^2} |\langle j_+, \vec{p} | D^+(0) | C \rangle|^2, \quad (8)$$

where we have decomposed the sum over  $j$  into two classes of states: those with isospin one

---

<sup>2</sup>The case where  $\langle \bar{q}q \rangle_C = 0$  is interesting in its own right. This problem is discussed in detail in ref. [4]

unit more ( $j_+$ ) or less ( $j_-$ ) than in  $|C\rangle$ . Equation (7) may be decomposed similarly, giving

$$- \langle \bar{q}q \rangle_C = \sum_{j_-} \frac{\bar{m}}{2|E_{j_-}|E_{j_-}} |\langle j_-, \vec{p} | D^-(0) | C \rangle|^2 + \sum_{j_+} \frac{\bar{m}}{2|E_{j_+}|E_{j_+}} |\langle j_+, \vec{p} | D^+(0) | C \rangle|^2 . \quad (9)$$

Now consider the sum rule given in Eq. (8) as one approaches the chiral limit. In isovector matter, the left hand side of this sum rule is nonzero, by definition. Accordingly, at least one term in this sum rule must be nonzero. However, all terms in the sum rule go as  $\bar{m}/E_j^3$  times matrix elements. Since the matrix elements are assumed to be finite in the chiral limit, one sees that the only way any term can make a contribution in the chiral limit is if  $\dim_\chi(E_j) \geq 2/3$ . This completes the demonstration of the first point of this Letter. Note, that this relation is written as inequality rather than an equality since it is possible that the chiral dimension of the matrix element could well be  $> 0$ . Indeed, as we will show below, given reasonable assumptions it is in fact  $> 0$ .

To proceed further we need to make the assumption that the chiral dimension of the isovector chemical potential  $\dim_\chi(\mu_{I=1}) = 0$ . Since we are implicitly taking the chiral limit at a fixed isospin density  $\rho_{I=1}$ , we can treat  $\rho_{I=1}$  as an *external parameter*, independent of the chiral parameter — the assumption which we must make is that the difference in energy density between this state and the the lowest isospin symmetric state is independent of  $\bar{m}$ . Although we cannot prove that  $\dim_\chi(\mu_{I=1}) = 0$  from first principles, we can present the following physical argument in its favor: The isovector interaction has a contribution produced by the  $\rho$ -meson exchange. Suppose we place an object of isospin  $I_3$  in the isovector medium. The interaction is  $(g_\rho^2/m_\rho^2) \rho_{I=1} I_3$ , and the corresponding chemical potential is  $(g_\rho^2/m_\rho^2) \rho_{I=1}$ . Here  $g_\rho$  and  $m_\rho$  are  $\rho$ -meson coupling constant and mass in the medium. Their chiral dimension in the vacuum is 0, and, unless something very unusual happens, this is also true in medium. Therefore  $\rho$ -exchange produces  $\dim_\chi(\mu_{I=1}) = 0$ . It is very unlikely that this result could be altered by other processes — they would have to exactly cancel the  $\rho$ -exchange mechanism.

Equipped with the assumption that  $\dim_\chi(\mu_{I=1}) = 0$ , we can continue the analysis of the sum rules (8) and (9). Suppose for definiteness that  $\mu_{I=1} > 0$  (as it is for neutron matter).

From definition of the chemical potential as the minimum energy needed to lower the isospin by one unit, we have

$$E_{j_{\mp}} = \mu_{I=1} \pm \delta E_{j_{\mp}} \quad , \quad \text{with } \delta E_{j_{\mp}} \geq 0 . \quad (10)$$

Therefore the energies of states with one unit of isospin less than  $|C\rangle$ ,  $E_{j_-}$ , can never go to zero. In the chiral limit only the states with  $E_j \rightarrow 0$  can contribute to the sum rules (8) and (9), and as a result only the states  $j_+$  contribute. Note, this is very different from the vacuum case — it shows that in neutron matter the  $\pi^+$  modes can be pseudo-Goldstone modes while the  $\pi^-$  cannot. Restricting the sum rules to the  $j^+$  modes only gives

$$\rho_{I=1} = \lim_{\bar{m} \rightarrow 0} \left( - \sum_{j_+} \frac{\bar{m}^2}{|E_{j_+}|^3} |\langle j_+, \vec{p} | D^+(0) | C \rangle|^2 \right) , \quad (11)$$

and

$$\lim_{\bar{m} \rightarrow 0} (\langle \bar{q}q \rangle_C) = - \lim_{\bar{m} \rightarrow 0} \left( \sum_{j_+} \frac{\bar{m}}{2|E_{j_+}|E_{j_+}} |\langle j_+, \vec{p} | D^+(0) | C \rangle|^2 \right) . \quad (12)$$

Now, the sum rule (11) contains only semi-negative contributions. Comparing the chiral dimensions on both sides, it follows that there must exist a mode for which

$$0 = 2 - 3 \dim_{\chi}(E_{j_+}) + 2 \dim_{\chi}(\langle j_+, \vec{p} = 0 | D^+(0) | C \rangle) . \quad (13)$$

The sum rule (12) may contain both positive and negative contributions, since the sign of  $E_{j_+}$  is not restricted. That means that in principle there may be cancellations of the leading chiral dimension on the RHS of Eq. (12), and we obtain the following inequality:

$$\dim_{\chi}(\bar{q}q) \geq 1 - 2 \dim_{\chi}(E_{j_+}) + 2 \dim_{\chi}(\langle j_+, \vec{p} = 0 | D^+(0) | C \rangle) . \quad (14)$$

The conjunction of conditions (13) and (14) gives  $\dim_{\chi}(E_{j_+}) \leq 1 + \dim_{\chi}(\langle \bar{q}q \rangle_C)$ . The inequality becomes equality unless there is an exactly cancellation of the leading order contribution on the RHS of Eq. (12). For instance, the equality is automatically the case if there is only one state  $|j_+, \vec{p} = 0\rangle$  which becomes a pseudo-Goldstone mode. Also, potential cancellations are not associated with any symmetry, hence it is difficult to imagine that the



leading chiral dimension can indeed be exact canceled on the RHS of Eq. (12). With no cancellations we have

$$\dim_{\chi}(E_{j_+}) = 1 + \dim_{\chi}(\langle \bar{q}q \rangle_C) . \quad (15)$$

If we are in the spontaneously broken phase, then we expect that  $\dim(\langle \bar{q}q \rangle_C) = 0$ , as in the vacuum. Clearly, this dimension has at least to be positive semi-definite, if the chiral condensate is to be well-behaved in the chiral limit. This leads to the following final result:

$$\dim_{\chi}(E_{j_+}) = 1 , \quad \dim_{\chi}(D^+) = 1/2 , \quad \dim_{\chi}(J_{5,0}^+) = 1/2 , \quad (16)$$

where the third equality follows from Eq. (6). Note that this behavior is radically different than in the vacuum, where we have

$$\dim_{\chi}(m_{\pi}) = 1/2 , \quad \dim_{\chi}(D^+) = 0 , \quad \dim_{\chi}(J_{5,0}^+) = 1/2 . \quad (17)$$

Of the three pionic pseudo-Goldstone modes in the vacuum, only 2 survive as pseudo-Goldstone modes in dense isovector matter. For negative  $\rho_{I=1}$  the negative charge excitation disappeared, the positive charge excitation became unusually soft,  $E_{\pi^+} \sim \overline{m}$ , and the neutral excitation retained its chiral dimension,  $E_{\pi^0} \sim \sqrt{\overline{m}}$ . If  $\rho_{I=1} > 0$ , then the positive and negative excitation change the roles.

The fact that only one member of the pair of charged pseudo-Goldstone mesons remains a pseudo-Goldstone mode in dense isospin-asymmetric matter is quite natural. The medium itself breaks the symmetry between the members of the pair — thus one would be quite surprised if they had the same energy. One expects the symmetry breaking induced the medium to split the degeneracy between the two states. However, as one approaches the chiral limit a pseudo-Goldstone mode must go to zero excitation energy. Thus, if both modes were pseudo-Goldstone modes they would both have to approach zero and hence would become degenerate.

Before giving an illustrative example, let us recapitulate our derivation, listing all assumptions made on the way. First, let us stress that what we were after is a fundamental result derived without any explicit reference to dynamics, microscopic structure of the

modes, *etc.* In matter these modes are undoubtedly quite complicated, involving particle-hole excitations, *etc.* We have not made any dynamical assumptions. These were our key ingredients:

1. The chiral limit is taken first, while the isovector density is kept constant.
2. “Reasonableness” conditions are that expectation values of operators, *e.g.*  $\langle \bar{q}q \rangle_C$ , do not diverge in the chiral limit. If this were not true, than the chiral limit would not make sense in the isovector matter.
3. The isovector chemical potential is assumed to be nonzero in the chiral limit.
4. We also assumed that there are no exact cancellations of the leading chiral powers in the sum rule (9). This assumptions is equivalent to having no (accidental) cancellations from a priori possible multiple branches of pseudo-Goldstone modes. If there is only one such branch, this result follows trivially.

Assumptions 1) and 2) are sufficient to show that there exists a pseudo-Goldstone mode for which  $\dim_\chi(E_j) \geq 2/3$ . Assumptions 1) – 4) give  $\dim_\chi(E_j) = 1$  for this mode.

Now, let us present a simple model which will illustrate the behavior of pseudo-Goldstone modes in nonsymmetric medium. Consider the pion propagating in an isovector medium and interacting with it through the  $\rho$ -meson exchange. Moreover, in this toy model, we will assume that the  $\rho$ -meson exchange is the *only* interaction between the pions and the medium. As is well known [5], if all vector-isovector interactions are mediated by the  $\rho$  meson, then consistency of the  $\rho$  exchange picture with the various soft-pion theorems requires the KSFR [6] relation  $m_\rho^2/g_\rho^2 = 2F_\pi^2$  to be exact. The inverse propagator for the  $\pi^\pm$  mesons with four-momentum  $p$  is  $(G(p)^\pm)^{-1} = p^2 - m_\pi^2 \mp p_0 A$ , where  $A = (g_\rho^2/m_\rho^2)\rho_{I=1}$ . The (positive energy) poles of  $G(p)$  for the pion at rest ( $\vec{p} = 0$ ) occur at

$$\begin{aligned} E^+ &= -A/2 + \sqrt{A^2/4 + m_\pi^2} \\ E^- &= A/2 + \sqrt{A^2/4 + m_\pi^2} \end{aligned} \tag{18}$$

In the chiral limit,  $E^+ \rightarrow m_\pi^2/A$  — this is the unusually soft pseudo-Goldstone mode, with  $\dim(E^+) = 1$ , and  $E^- \rightarrow A$  — this is no longer a pseudo-Goldstone mode, since  $\dim(E^-) = 0$ . This is precisely the behavior which we predicted. Obviously, the neutral pion is unchanged by the isovector interactions, and  $\dim_\chi(E^+) = 1/2$ .

Let us also examine the matrix elements of the operator  $D^\pm$ . We have, from definition (5),  $D^\pm = \partial^\mu J_{5,\mu}^\pm / \overline{m} = (F_\pi m_\pi^2 / \overline{m}) \pi^\pm$ , where  $\pi^\pm$  is the (charged) pion interpolating field, and  $F_\pi$  is the pion decay constant in the vacuum. Let abbreviate the states corresponding to excitations  $E^\pm$  by  $|\pm\rangle$ . Using  $|\langle \pm | \pi^\pm | C \rangle|^2 = \lim_{p_0 \rightarrow E^j} 2E^j (p_0 - E^j) G^\pm(p_0, \vec{p} = 0)$ , where  $j = +$  or  $-$ , we find that in the chiral limit  $|\langle + | D^+ | C \rangle|^2 \rightarrow 2F_\pi^2 m_\pi^6 / (A^2 \overline{m}^2) \sim \overline{m}$ , in compliance to the general result (13), and  $|\langle - | D^+ | C \rangle|^2 \sim 1$ . Only the positive charge mode saturates the sum rules (8) and (9). Explicitly, we get

$$\rho_{I=1} = F_\pi^2 A = 2 \left( \frac{g_\rho F_\pi}{m_\rho} \right)^2 \rho_{I=1} , \quad (19)$$

and

$$\lim_{\overline{m} \rightarrow 0} \langle \overline{q}q \rangle_C = - \frac{F_\pi^2 m_\pi^2}{\overline{m}} . \quad (20)$$

Equation (19) means that, as expected, consistency in the chiral limit requires the KSFR relation to be exact. Relation (20) also shows the formal consistency, since exactly the same equation is obtained by considering the sum rule for the neutral pion. Since the neutral pion is not affected by the  $\rho$ -exchange, we immediately get Eq. (20).

Under what circumstances is the analysis here useful? — *i.e.* at what isospin density does the system go from effectively having three pseudo-Goldstone modes whose energies all have chiral dimension  $1/2$  to having two pseudo-Goldstone modes whose energies have chiral dimension of  $1$  and  $1/2$ . Formally, the analysis is valid for any nonzero isospin density *provided we go to the  $m_q \rightarrow 0$  limit*. On the other hand, in nuclear physics the value of  $m_q$ , though small, is nonzero. Thus, the question of interest is under what circumstances is the quark mass small enough so that the results of  $m_q \rightarrow 0$  limit are close to the physical results. The toy model considered above, although probably quite unrealistic, gives us considerable insight.

Consider Eq. (18); one approaches the limit in which the present analysis applies when  $|A|/2 \gg m_\pi$ . We note that in this toy model  $A = (\rho_{I=1} g_\rho^2)/m_\rho^2$  is the chemical potential associated with isospin,  $\mu_{I=1}$ . Also in this simple model,  $m_\pi$  is the energy of the pseudo-Goldstone mode when  $\rho_{I=1} = 0$ . In the general case it may be different from  $m_\pi$ , so let us denote it by  $E_0$ . Thus, a reasonable general criterion for the region of applicability of our analysis is when  $\mu_{I=1} \gg E_0$ . In our toy model, this condition is equivalent to  $|\rho_{I=1}| \gg \bar{\rho} \equiv 4F_\pi^2 m_\pi \simeq 3.7\rho_0$ , where  $\rho_0$  is the nuclear saturation density. The above estimate used the vacuum values of  $F_\pi$  and  $m_\pi$ . However, this probably leads to an overestimation of  $\bar{\rho}$ , because matter with large isovector density also has large baryon density,  $\rho_B$ , which considerably reduces the value of  $F_\pi$  from its vacuum value [7–14]. In turn,  $\bar{\rho}$  should be substantially reduced by the presence of  $\rho_B$ .

We also note one more fact illustrated by our toy model. It is apparent that the chiral limit and the low-isovector-density limit do not commute. In our model in the chiral limit the expansion parameter is  $\alpha = m_\pi^2 F_\pi / \rho_{I=1}$  — this is singular in the isovector density. Conversely, in the low density limit the expansion parameter is  $1/\alpha$ , and this, in turn, is singular in  $m_\pi$ . We believe this is a manifestation of a general result.

Now let us turn our attention to kaons, since recently the possibility of  $S$ -wave kaon condensation has been extensively discussed [15–20,14]. The meaning of the chiral limit in this case is somewhat subtle and this subtlety greatly affects the applicability of our analysis. As the current quark masses tend to zero (including  $m_s$ ) two phenomena happen in the nuclear medium. One is the change of properties of particles, which is the subject of our analysis. The second phenomenon is the change of the ground state of the medium itself.

In the strict  $SU(3)$  chiral limit the ground state of matter has equal amount of up, down and strange quarks. In that case the octet of axial vector charges commutes with the (baryon number) constraint, and pions, kaons and  $\eta$  are the usual pseudo-Goldstone excitations with  $E^a \sim \sqrt{m_q}$ . This is not the situation of relevance at moderately low densities. At first sight, one might rather keep the matter non-strange, by imposing the

constraint  $\mu_u u^+ u + \mu_d d^+ d$ , and watch the formal behavior of the kaon as  $m_s \rightarrow 0$ . In this case the  $a = 4, \dots, 7$  components of the axial vector charge do not commute with the constraint. The situation is analogous to the case of the charged pion in nonsymmetric medium. We only have to change the usual I-spin into U-spin or V-spin. Again, only two kaon-like excitation (out of 4 in the vacuum) are pseudo-Goldstone modes, with excitation energy  $\sim m_q$ .

The preceding argument, however, is formal; it depends on using  $\mu_u u^+ u + \mu_d d^+ d$  as the constraint. This is not the relevant constraint for the neutron matter calculations—at least in the interesting applications to neutron stars. In that case of interest the constraints are on the net baryon density and the net charge density [15–19]. The reason that these are the appropriate constraints is clear—in the neutron star case one is working at long times scales for which the system can be expected to be in equilibrium with respect to weak interaction processes which can change the flavor quantum numbers. Accordingly, the analysis given above does not give any quantitative insight into the problem of kaon condensation.

In conclusion, we have shown the behavior of pseudo-Goldstone modes in dense isospin-asymmetric nuclear matter is very different from the case of the vacuum or isospin symmetric matter. Of the three pionic modes which exist in the vacuum only two remain in dense isovector matter. Moreover, of these remaining modes, the charged mode is anomalously light—having a chiral dimension of one rather than one half.

This work has been supported by the NSF–Polish Academy of Science grant #INT-9313988, NSF PYI grant #PHY-9058487, DOE grant #DE-FG02-40762, and by the Maria Skłodowska-Curie grant #PAA/NSF-94-158.

## REFERENCES

- [1] See for example T. E. O. Ericson and W. Weise, *Pions and Nuclei*, Oxford Science Pub. (1988).
- [2] M. Gell-Mann, R. Oakes and B. Renner, Phys. Rev. **175** (1968) 2195.
- [3] M. Lutz, A. Steiner and W. Weise, Nucl. Phys. **A 574** (1994) 755.
- [4] T. D. Cohen and W. Broniowski, U. of Maryland preprint DOE-ER-40762-043, hep-ph/9407354, to appear in Phys. Lett. B.
- [5] See for example R. K. Bhaduri, *Models of the Nucleon, from Quarks to Soliton*, Addison-Wesley (1988), Lecture Notes and Supplements in Physics.
- [6] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16** (1966) 255; Riazuddin and Fayazuddin, Phys. Rev. **147** (1966) 1071.
- [7] E. G. Drukarev and E. H. Levin, Nucl. Phys. **A 511** (1990) 679; **A516** (1990) 715(E); Prog. Part. Nuc. Phys. **27** (1991) 77.
- [8] T. D. Cohen, R. J. Furnstahl, D. K. Griegel, Phys. Rev. C **45** (1992) 1881; Phys. Rev. Lett. **67** (1991) 961.
- [9] M. Lutz, S. Klimt, W. Weise, Nucl. Phys. **A 542** (1992) 521.
- [10] L. S. Celenza, C. M. Shakin, W. Dong and X. Zhu, Phys. Rev. C **48** (1993) 159.
- [11] M. Ericson, Phys. Lett. **B301**, 11 (1993).
- [12] G. Chanfrey and M. Ericson, Nucl. Phys. **A556**, 427 (1993).
- [13] M. C. Birse and J. A. McGovern, Phys. Lett **B309**, 234 (1993).
- [14] M. C. Birse, University of Manchester preprint MC/TH 94/13, nucl-th/9406029.
- [15] D. Montano, H. D. Politzer and M. B. Wise, Nucl. Phys. **B375** (1992) 507.

- [16] H. D. Politzer and M. B. Wise, Phys. Lett. **B273** (1991) 156.
- [17] D. B. Kaplan and A. E. Nelson, Phys. Lett. **B175** (1986) 57; **B192** (1987) 193.
- [18] G. E. Brown, K. Kubodera, M. Rho, V. Thorsson, Phys. Lett. **B291** (1992) 355.
- [19] G. E. Brown, C.-H. Lee, M. Rho, V. Thorsson, Nucl. Phys. **A567** (1994) 937.
- [20] H. Yabu, F. Myhrer, K. Kubodera, Phys. Rev. **D 50** (1994) 3549.